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Stable Model Semantics in ProbLog and its Applications in Argumentation

DTAI Seminar

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KU Leuven
February 21, 2022

0 Overview

This presentation discusses the following topics:

- How to use Probabilistic Logic Programming (PLP) to model Probabilistic Argumentation problems

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- How to use Probabilistic Logic Programming (PLP) to model Probabilistic Argumentation problems
- How to expand traditional PLP semantics to reason over such models
- How to implement a PLP framework for the new semantics

0 Overview

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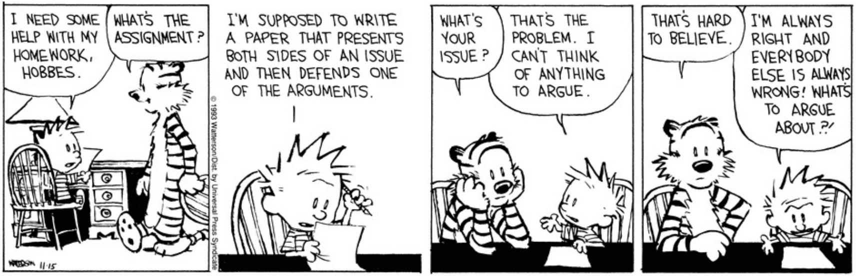
1 Outline

1 What is argumentation?

"Humans argue."[1]

[1] Either you already believe it or you would need to argue against it. Atkinson et al., "Towards Artificial Argumentation", *AI Mag.* 38.3 (2017)



1 What is argumentation?

Argumentation has two main components:

1. Argumentation mining:
   - Find - Sources (newspapers, journals,. . .)
   - Extract - Relevant text (argumentative/non-argumentative)
   - Classify - Arguments (evidence, claims, proponent/opponent . . .)

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We focus on modelling and reasoning. . .

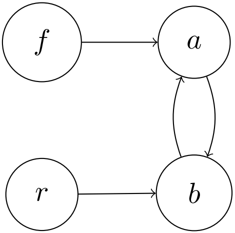
2 Outline

2 Model: Abstract Argumentation Frameworks

Definition

An Abstract Argumentation Framework (AAF) is a pair $(Args, Att)$ where $Args$ is a set of arguments and Att is a binary relation over $Args$: $Att \subseteq Args \times Args$.

We can represent an AAF as a directed graph:



f = "Hotel B is recommended by a friend"
a = "Book hotel A"
r = "Hotel A has higher review score"
b = "Book hotel B"
$Args = \{f, a, r, b\}$
$Att = \{(f, a), (a, b), (b, a), (r, b)\}$

2 Reasoning: Acceptability semantics

Given an AAF, there are different "recipies" to determine acceptable arguments[1]. A set of arguments $A \subseteq Args$ can be:

- Conflict-free (cf.) - when there are no $a, b \in A$ s.t. $(a, b) \in Att$

[1] Baroni, Caminada, and Giacomin, "An introduction to argumentation semantics", *Knowl. Eng. Rev.* 26.4 (2011).

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Each extension to the original AAF requires new ad-hoc semantics to define classes of acceptable arguments in the new framework.

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Combining models

Integrating different formalisms becomes difficult: it requires to redefine the acceptability semantics with new concepts tailored to the specific combination of modelling strategies.

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Combining models

Integrating different formalisms becomes difficult: it requires to redefine the acceptability semantics with new concepts tailored to the specific combination of modelling strategies.

What if we had a general-purpose framework with powerful semantics for encoding many extensions?

3 Outline

3 Modelling: the advantages of Probabilistic Logic Programming

Why Probabilistic Logic Programming

Probabilistic Logic Programming and its tools offer a *general purpose* framework for logical reasoning and uncertainty. This gives some advantages over traditional argumentation frameworks:

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- Flexibility and modularity - change and evolve a model for specific domains without changing the underlying reasoning algorithms.

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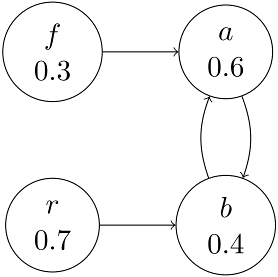
Probabilistic Logic Programming and its tools offer a *general purpose* framework for logical reasoning and uncertainty. This gives some advantages over traditional argumentation frameworks:

- Succinctness and expressivity - express complex interactions between random variables with (first-order) logic rules.
- Flexibility and modularity - change and evolve a model for specific domains without changing the underlying reasoning algorithms.
- PLP tools - bring to argumentation the general suite of PLP inference and learning algorithms.

3 Modelling: What about probabilities?

Definition

A *probabilistic argument graph* (or probabilistic AAF) is a tuple $(Args, Att, P^*)$ where $(Args, Att)$ is an AAF and P^* is a function: $P^* : Args \rightarrow [0, 1]$.



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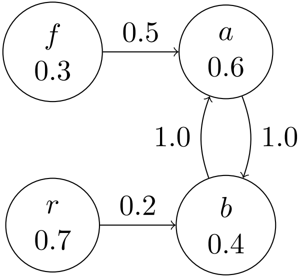
A *probabilistic argument graph* (or probabilistic AAF) is a tuple $(Args, Att, P^*)$ where $(Args, Att)$ is an AAF and P^* is a function: $P^* : Args \to [0,1]$. We can also add a function over attacks: $P^\times : Args \times Args \to [0,1]$.



3 Modelling: what do probabilities represent?

Probabilities as *degrees of belief*: they represent how much the argument or relation is believed.

Epistemic interpretation

A probabilistic argument graph describes:

- arguments' prior beliefs (P^*) i.e. a bias.

3 Modelling: what do probabilities represent?

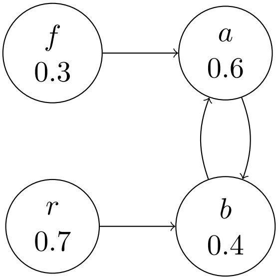
Probabilities as *degrees of belief*: they represent how much the argument or relation is believed.

Epistemic interpretation

A probabilistic argument graph describes:

- arguments' prior beliefs (P^*) i.e. a bias.
- how the initial bias is influenced by the other arguments.

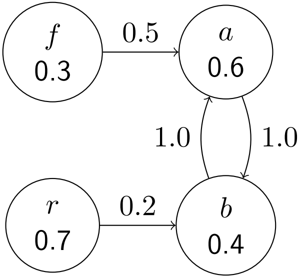
3 Modelling: from *probabilistic* AAFs to ProbLog




```
0.3::f. 0.6::a.
0.7::r. 0.4::b.
\+ a :- f.
\+ b :- r.
\+ b :- a.
\+ a :- b.
```

$$P(f) = 0.3 \ P(r) = 0.7$$
$$P(a) = 0.4 \ P(b) = 0.1$$

3 Modelling: from *gradual* AAFs to ProbLog




```
0.3::f. 0.6::a.
0.7::r. 0.4::b.
0.5:: \+ a :- f.
0.2:: \+ b :- r.
1.0:: \+ b :- a.
1.0:: \+ a :- b.
```

$$P(f) = 0.3 \; P(r) = 0.7$$
$$P(a) = 0.42 \; P(b) = 0.26$$

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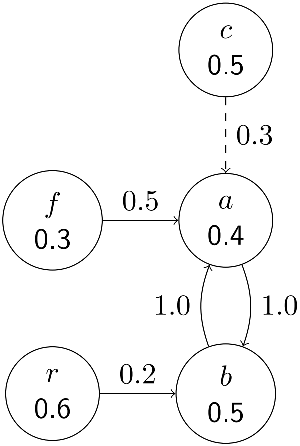
3 Modelling: from *bipolar* AAFs to ProbLog

c: "Hotel A has a nice view"

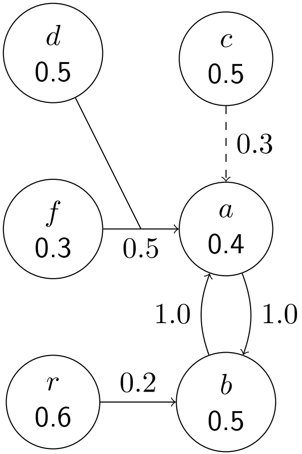



```
0.3::f. 0.6::a.
0.7::r. 0.4::b.
0.5::c.
0.5::\+ a :- f.
1.0::\+ b :- a.
1.0::\+ a :- b.
0.2::\+ b :- r.
0.3::a :- c.
```

$P(f) = 0.3$ $P(r) = 0.7$
$P(a) = 0.47$ $P(b) = 0.25$

3 Modelling: from *set-based* AAFs to ProbLog

d: "Same taste in hotels"




```
0.3::f. 0.6::a.
0.7::r. 0.4::b.
0.5::c. 0.5::d.
0.5::\+ a :- f, d.
1.0::\+ b :- a.
1.0::\+ a :- b.
0.2::\+ b :- r.
0.3::a :- c.
```

$P(f) = 0.3$ $P(r) = 0.7$
$P(a) = 0.5$ $P(b) = 0.24$

4 Outline

4 Probabilistic Logic Programs

Distribution semantics

Each possible choice of all probabilistic facts (total choice) defines a *possible world*: a deterministic program of the chosen facts plus logic rules

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Assumption of traditional PLP semantics

All choices are modelled by probabilistic facts: the total choice uniquely determines the truth value of all atoms.

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All choices are modelled by probabilistic facts: the total choice uniquely determines the truth value of all atoms.

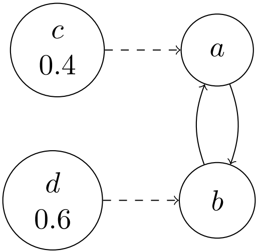
Example

In our hotel example for the *probabilistic* choice where neither the friend nor the reviews influence the choice of the hotel, logic still prescribes a choice between the two options.

4 Probabilistic Logic Programs

Distribution semantics + Well-founded model semantics

Traditional PLP frameworks cannot reason over programs with cyclic dependencies through negation

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b :- d.
a :- \+ b.
b :- \+ a.
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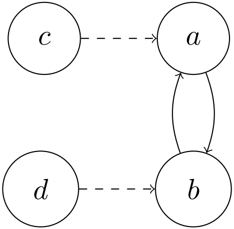
4 Probabilistic Logic Programs

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c.
d.
a :- c.
b :- d.
a :- \+ b.
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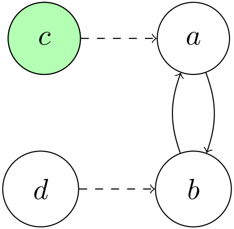
Possible world $\omega_1 = \{c, d\}$: $P(\omega_1) = 0.4 \cdot 0.6 = 0.24$

4 Probabilistic Logic Programs

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Possible world $\omega_1 = \{c, d\}$: $P(\omega_1) = 0.4 \cdot 0.6 = 0.24$

$\textsc{mod}(\omega_1) = \{a, b, c, d\}$

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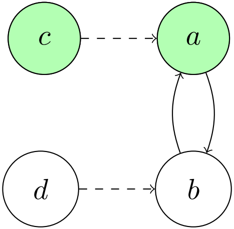
4 Probabilistic Logic Programs

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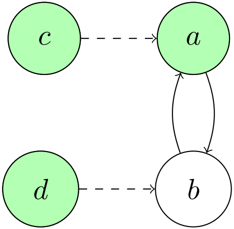


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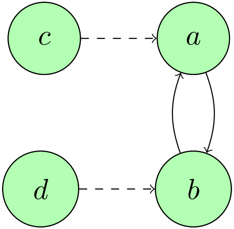


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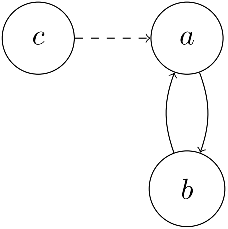
4 Probabilistic Logic Programs

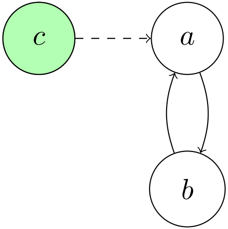



```
c.

a :- c.
b :- d.
a :- \+ b.
b :- \+ a.
```

Possible world $\omega_2 = \{c\}$: $P(\omega_2) = 0.4 \cdot (1 - 0.6) = 0.16$

4 Probabilistic Logic Programs




```
c.

a :- c.
b :- d.
a :- \+ b.
b :- \+ a.
```

Possible world $\omega_2 = \{c\}$: $P(\omega_2) = 0.4 \cdot (1 - 0.6) = 0.16$

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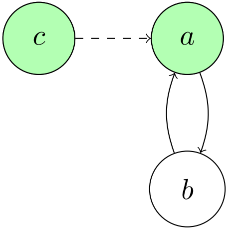
4 Probabilistic Logic Programs




```
c.

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b :- d.
a :- \+ b.
b :- \+ a.
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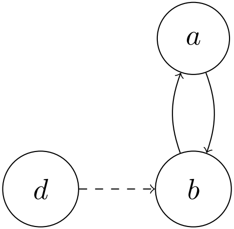
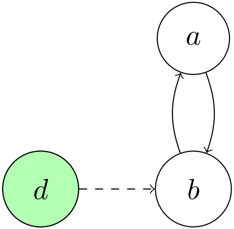
4 Probabilistic Logic Programs




```
d.
a :- c.
b :- d.
a :- \+ b.
b :- \+ a.
```

Possible world $\omega_3 = \{d\}$: $P(\omega_3) = (1 - 0.4) \cdot 0.6 = 0.36$

4 Probabilistic Logic Programs

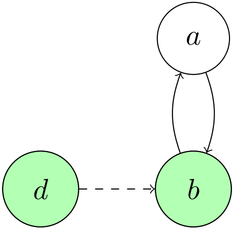



```
d.
a :- c.
b :- d.
a :- \+ b.
b :- \+ a.
```

Possible world $\omega_3 = \{d\}$: $P(\omega_3) = (1 - 0.4) \cdot 0.6 = 0.36$

$\text{MOD}(\omega_3) = \{b, d\}$

4 Probabilistic Logic Programs

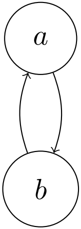



```
d.
a :- c.
b :- d.
a :- \+ b.
b :- \+ a.
```

Possible world $\omega_3 = \{d\}$: $P(\omega_3) = (1 - 0.4) \cdot 0.6 = 0.36$

$\text{MOD}(\omega_3) = \{b, d\}$

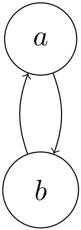
4 Probabilistic Logic Programs




```
  a :- c.
  b :- d.
a :- \+ b.
b :- \+ a.
```

Possible world $\omega_4 = \{\}$: $P(\omega_4) = (1 - 0.4) \cdot (1 - 0.6) = 0.24$

4 Probabilistic Logic Programs




```
  a :- c.
  b :- d.
→ a :- \+ b.
→ b :- \+ a.
```

Possible world $\omega_4 = \{\}$: $P(\omega_4) = (1 - 0.4) \cdot (1 - 0.6) = 0.24$

$\text{MOD}(\omega_4) = ?$

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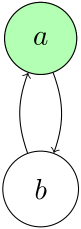
4 Probabilistic Logic Programs




```
    a :- c.
    b :- d.
a :- \+ b.
b :- \+ a.
```

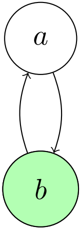
$\top \leftarrow \top$

$\bot \leftarrow \bot$

Possible world $\omega_4 = \{\}$: $P(\omega_4) = (1 - 0.4) \cdot (1 - 0.6) = 0.24$

$\text{MOD}(\omega_4) = \{a\}$

4 Probabilistic Logic Programs




```
      a :- c.
      b :- d.
a :- \+ b.
b :- \+ a.
```

$\bot \leftarrow \bot$

$\top \leftarrow \top$

Possible world $\omega_4 = \{\}$: $P(\omega_4) = (1 - 0.4) \cdot (1 - 0.6) = 0.24$

$\text{MOD}(\omega_4) = \{a\}$ *or* $\{b\}$

4 Probabilistic Logic Programs: well-founded vs stable model semantics

- Valid ProbLog programs are programs where each total choice corresponds to *exactly one* well-founded model.

[2] Totis, Kimmig, and De Raedt, "SMProbLog: Stable Model Semantics in ProbLog and its Applications in Argumentation", *StarAI* abs/2110.01990 (2021).

4 Probabilistic Logic Programs: well-founded vs stable model semantics

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- ω_4 has no well-founded model.

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SMProbLog programs

SMProbLog[2] reasons over the (zero or more) *stable models* of each total choice.

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- ω_4 has no well-founded model.

SMProbLog programs

SMProbLog[2] reasons over the (zero or more) *stable models* of each total choice.

If a program has one well-founded model then it is its unique stable model

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4 Probabilistic Logic Programs: success probability

P.W.	Facts	Probability	Model
ω_1	$\{c, d\}$	0.24	$\{a, b, c, d\}$
ω_2	$\{c\}$	0.16	$\{c, a\}$
ω_3	$\{d\}$	0.36	$\{d, b\}$
ω_4	$\{\}$	0.24	$\{a\}, \{b\}$

```
0.4::c.
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P.W.	Facts	Probability	Model
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ω_3	$\{d\}$	0.36	$\{d, b\}$
ω_4	$\{\}$	0.24	$\{a\}, \{b\}$

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What about the probability of atoms?

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P.W.	Facts	Probability	Model
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ω_4	$\{\}$	0.24	$\{a\}, \{b\}$

```
0.4::c.
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a :- c.
b :- d.
a :- \+ b.
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```

What about the probability of atoms?

$$P(a) = \sum_{M \models a, M = \text{MOD}(\omega)} P(M)$$

4 Probabilistic Logic Programs: success probability

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What about the probability of atoms?

$$P(a) = \sum_{M \models a, M = \textsc{mod}(\omega)} P(M)$$

We need to define the probability distribution of the models...

4 Reasoning over non-probabilistic choices

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4 Reasoning over non-probabilistic choices

- With stable model semantics we have choices that are prescribed by logical consistency rather than probabilistic facts.
- We thus assume that for a fixed possible world all further (logical) choices are equally possible.
- The probability of a model is thus the probability of the possible world normalized w.r.t. the number of non-probabilistic choices.

4 Reasoning over non-probabilistic choices

Let $\mathcal{L}$ be a probabilistic logic program, $\Omega_{\mathcal{L}}$ its set of possible worlds, and $\textsc{mod}(\omega)$ the set of models of a possible world $\omega \in \Omega_{\mathcal{L}}$, then:

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We thus solve for a query φ:

$$\widehat{WMC}_{\mathcal{L}}(\varphi) = \sum_{M \models \varphi, M \in \textsc{mod}(\omega), \omega \in \Omega_{\mathcal{L}}} \frac{1}{|\textsc{mod}(\omega)|} \cdot \prod_{l \in M} w(l)$$

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ω_4	$\{\}$	0.24	$\{a\}$, $\{b\}$	0.12, 0.12

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4 Probabilistic Logic Programs: inconsistencies

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Even/odd cycles through negations

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  Lin and Zhao, "On Odd and Even Cycles in Normal Logic Programs" (2004)

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We use a *three-valued interpretation* to keep track of the probability of inconsistent choices.

4 Probabilistic Logic Programs: inconsistencies

Three-valued interpretaiton

A three-valued interpretation is a pair (T, F) where T is the set of atoms interpreted *true* and F is the set of atoms interpreted *false*. The atoms outside $T \cup F$ are interpreted as *inconsistent*.

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$$P(\mathcal{L} \models \bot) = 0.24 + 0.16 + 0.36 = 0.76$$

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5 Outline

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5 Pipelines

ProbLog

Program → Grounding → [Cycle breaking → CNF] → Knowledge compilation → Circuit → WMC

SMProbLog

Program → Grounding → Knowledge compilation → Circuit → [Stable models → $\widehat{WMC}$]

5 Pipelines: differences

- Cycle breaking and CNF conversion do not preserve stable models

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5 Pipelines: differences

- Cycle breaking and CNF conversion do not preserve stable models
- We need a knowledge compiler for stable models: DSHARP[3]
- We need then to count the stable models to solve the $\widehat{WMC}$ problem

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5 Pipelines: WMC

Definition

A *NNF* is a rooted directed acyclic graph in which each leaf node is labeled with a literal and each internal node is labeled with a disjunction or conjunction. A smooth *d-DNNF* is an NNF with the following properties:

- Deterministic: for all disjunctive nodes the children represent formulas pairwise inconsistent.
- Decomposable: the subtrees rooted in two children of a conjunction node do not have atoms in common.
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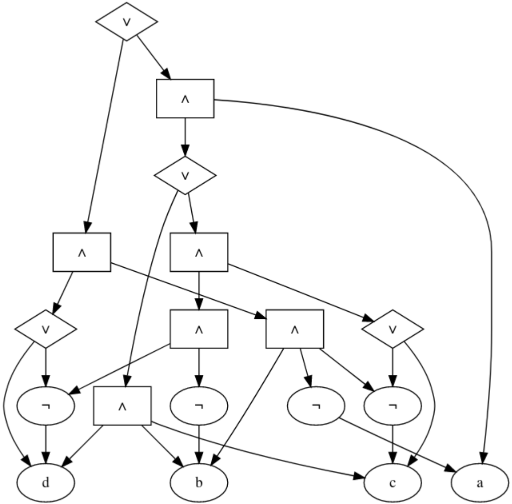
WMC can be solved in polynomial time on d-DNNFs

5 Pipelines: WMC



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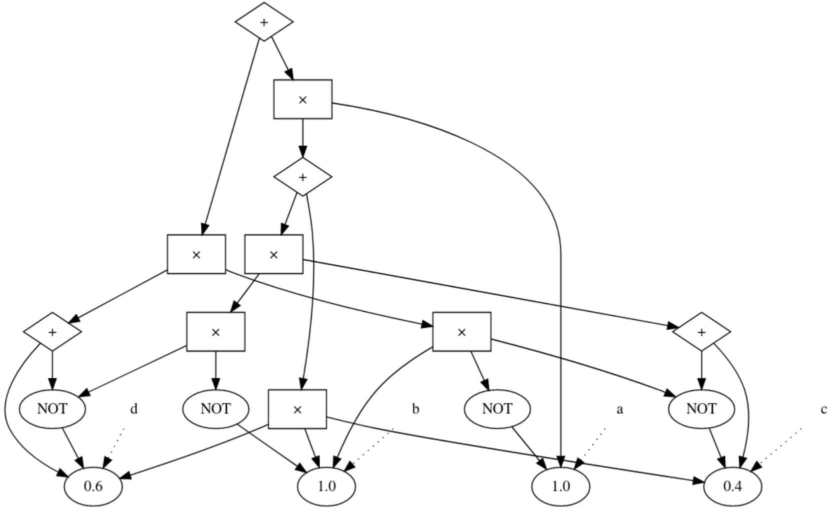

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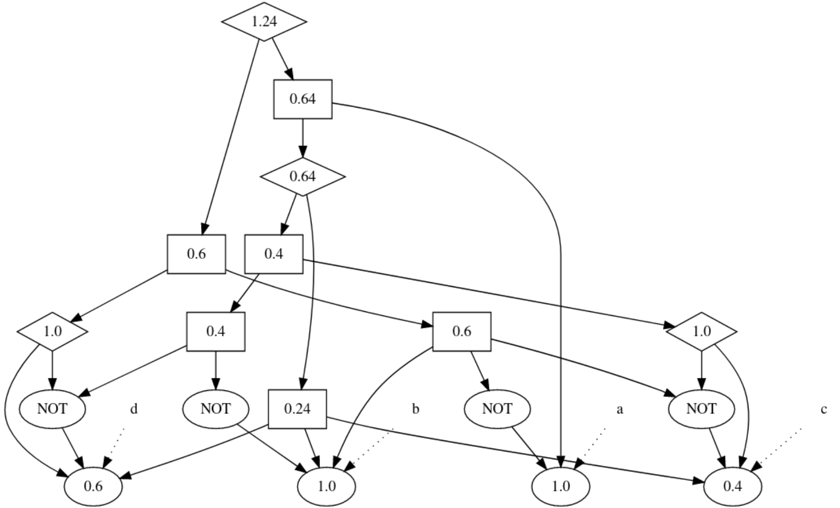

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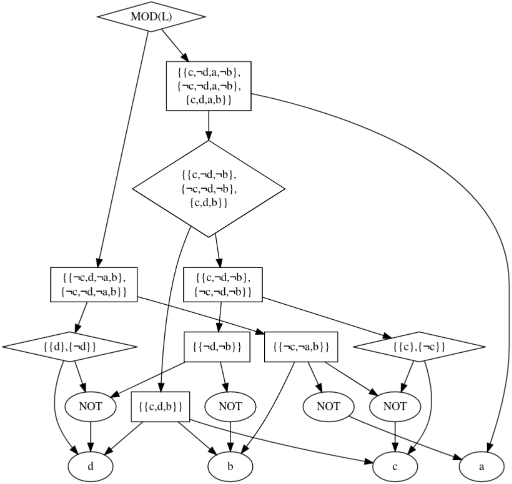

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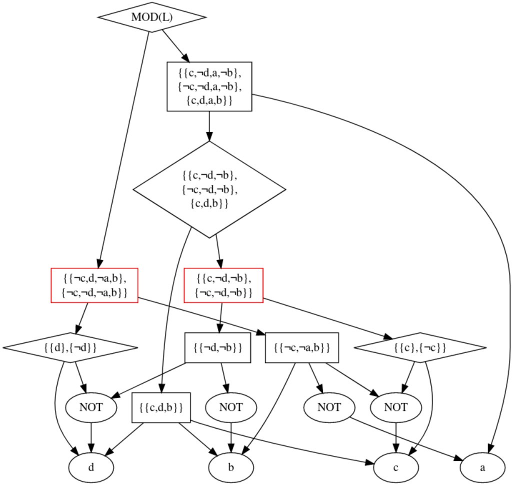

5 Pipelines: from WMC to $\widehat{WMC}$

We are solving two tasks in one circuit[4]:

[4]Kiesel, Totis, and Kimmig, "Efficient Knowledge Compilation Beyond Weighted Model Counting", *(Under review)* (2022).

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Constrained Knowledge Compilation allows us to solve the two problems in polynomial time by constraining the order in which the variables are decided.

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Trade-off

Constrained variable orders typically lead to an increase of the size of the circuit.

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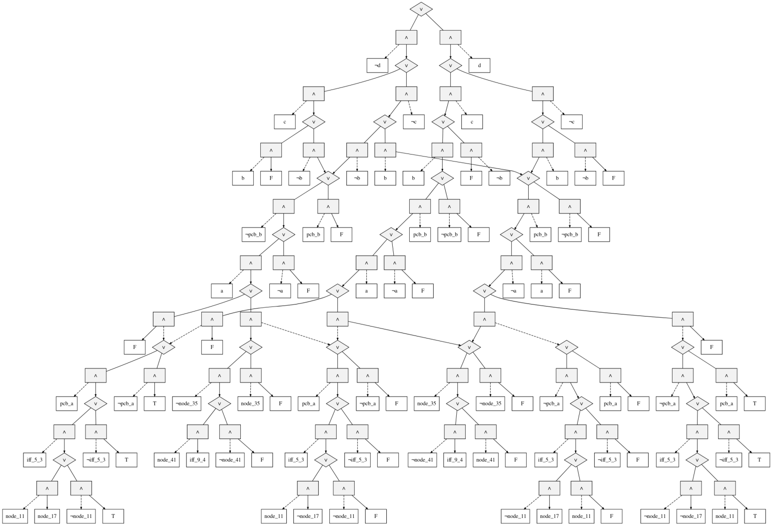
5 Pipelines: constrained compilation



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6 Outline

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6 Summary

- Argumentation frameworks come in many flavors and interpretations: we combine with PLP the most important ones, unlocking additional resources for inference and learning in argumentation.

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- Argumentation frameworks come in many flavors and interpretations: we combine with PLP the most important ones, unlocking additional resources for inference and learning in argumentation.
- Traditional PLP semantics do not support common logical patterns in argument graphs: we introduce more expressive semantics allowing us reason on such programs.
- New semantics require new inference methods: we resort to pipelines/compilers for stable model semantics.
- Not all circuits that allow efficient inference for WMC are suitable for $\widehat{WMC}$: we constrain the variable order to be able to solve efficiently two tasks in one circuit.

Questions?