Lifted Reasoning for Combinatorial Counting

Journal of Artificial Intelligence Research 76: 1-58 (2023)

Pietro Totis, Jesse Davis, Luc De Raedt, Angelika Kimmig

ECAI, 21 Oct. 2024

Combinatorics math problems

Example 1

“In how many different ways can the letters in B A N A N A be written?“

Example 2

“A shipment of 12 different TVs contains 3 defective ones. In how many ways can a hotel purchase 5 of these TVs and receive at least 2 of the defective ones?“

Fundamental components:

Combinatorics math problems

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Fundamental components:

- Multisets

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Combinatorics math problems

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Fundamental components:

- Multisets
- Configurations

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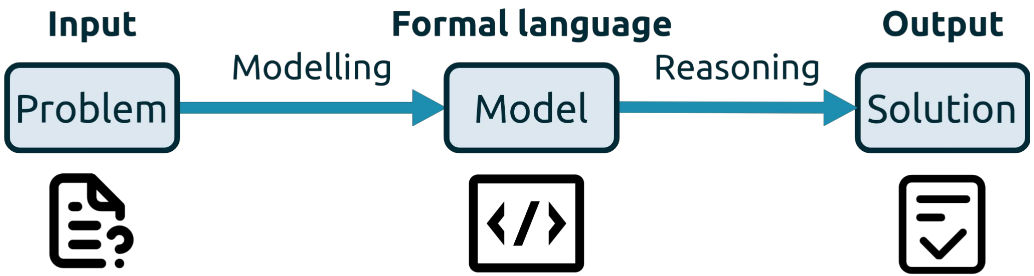
Fundamental components:

- Multisets
- Configurations
- Constraints

Combinatorial Problem Solving



Problem

Compute the solution of a combinatorics math problem: given a set of objects find the number of valid configurations defined.

Components

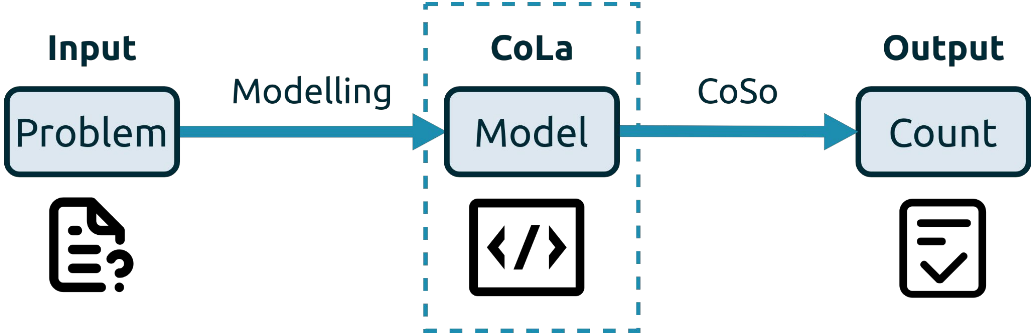
Key points to define:

1. Formal language
2. Reasoning

Output

The count of valid configurations (*model counting*).

Contribution 1: CoLa language



Research question
What are the key characteristics of a modelling language for encoding combinatorics math problems?

Key novelty
Generalize sets of objects (*constants*) to multisets in a declarative language.

Sets vs. multisets

Traditional declarative frameworks represent the objects of the domain with *distinguishable* constants (same constant = same object).

Example 1

"In how many different ways can the letters in B A N A N A be written?"

Set: { B A_1 N_1 A_2 N_2 A_3 }

Multiset: { B A N A N A }

Sets vs. multisets

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Example 1
"In how many different ways can the letters in B A N A N A be written?"

Set: { B A_1 N_1 A_2 N_2 A_3 }

Multiset: { B A N A N A }

Defining objects with constants that are always distinguishable does not affect *satisfiability* but does affect *counting.*

B A_1 N_1 A_2 N_2 A_3
B A_1 N_2 A_2 N_1 A_3

Count = 2 words

B A_1 N A_2 N A_3
B A_1 N A_2 N A_3

Count = 1 word

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Combinatorics math word problems

Example 1
"In how many different ways can the letters in B A N A N A be written?"

Result
60

Example 2
"A shipment of 12 different TVs contains 3 defective ones. In how many ways can a hotel purchase 5 of these TVs and receive at least 2 of the defective ones?"

Result
288

Contribution 1: CoLa language

Example 1

"In how many different ways can the letters in B A N A N A be written?"

Contribution 1

```
universe letters =
{a,a,a,n,n,b};
word in [letters];
#word = 6;
```

Result

60

Example 2

"A shipment of 12 different TVs contains 3 defective ones. In how many ways can a hotel purchase 5 of these TVs and receive at least 2 of the defective ones?"

Contribution 1

```
labelled property tvs;
property defective;
#tvs = 12;
#( tvs & defective ) = 3;
purchase in { tvs };
#purchase = 5;
#( purchase & defective )>=2;
```

Result

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Contribution 1: CoLa language - multisets

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"In how many different ways can the letters in B A N A N A be written?"

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Contribution 1: CoLa language - constraints

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Contribution 2

Result
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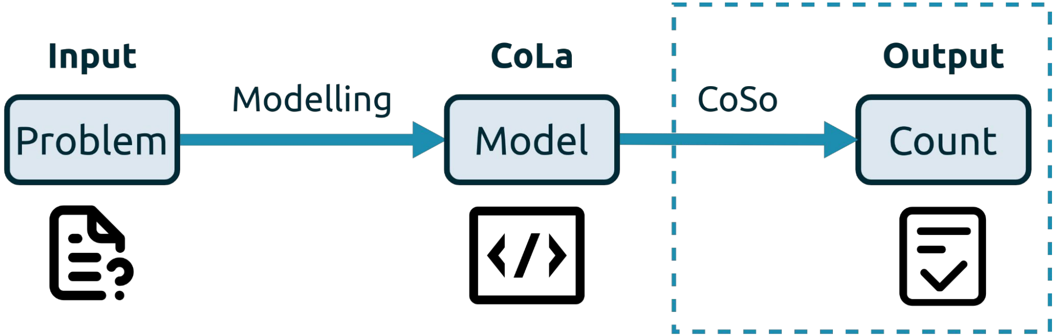
Contribution 2: CoSo



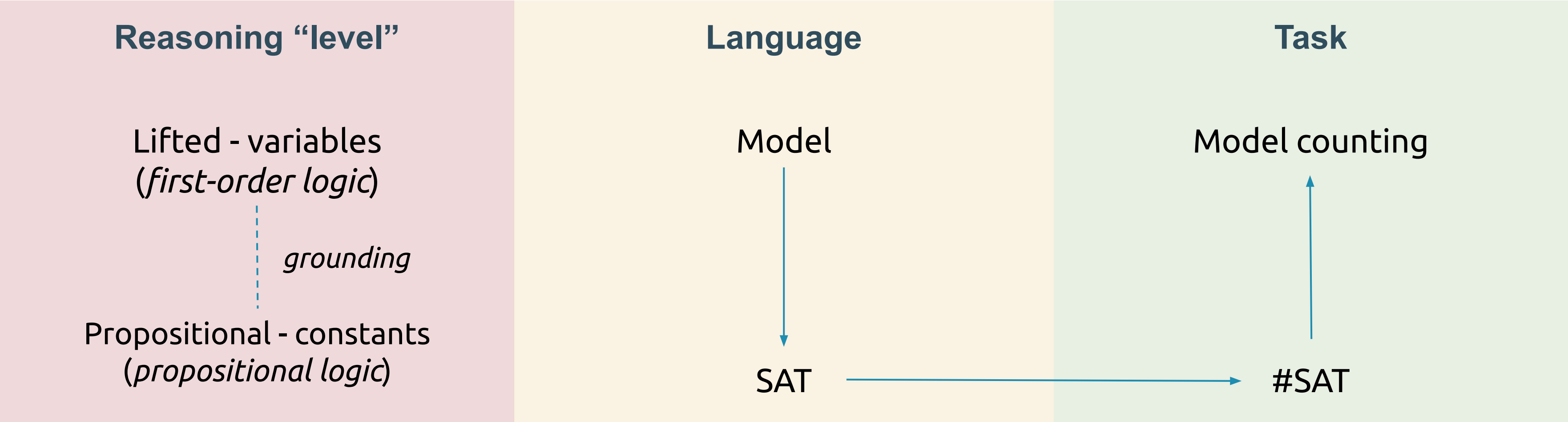
Research question

How can models for combinatorics math problems be efficiently solved?

Key novelty

Lifted reasoning: exploit the size of groups and symmetries of the problem to count efficiently.

Propositional reasoning



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Lifted reasoning

Reasoning "level"	Language	Task
Lifted - variables (*first-order logic*)	Model →	Model counting
⋮ *grounding*		
Propositional - constants (*propositional logic*)	SAT	#SAT

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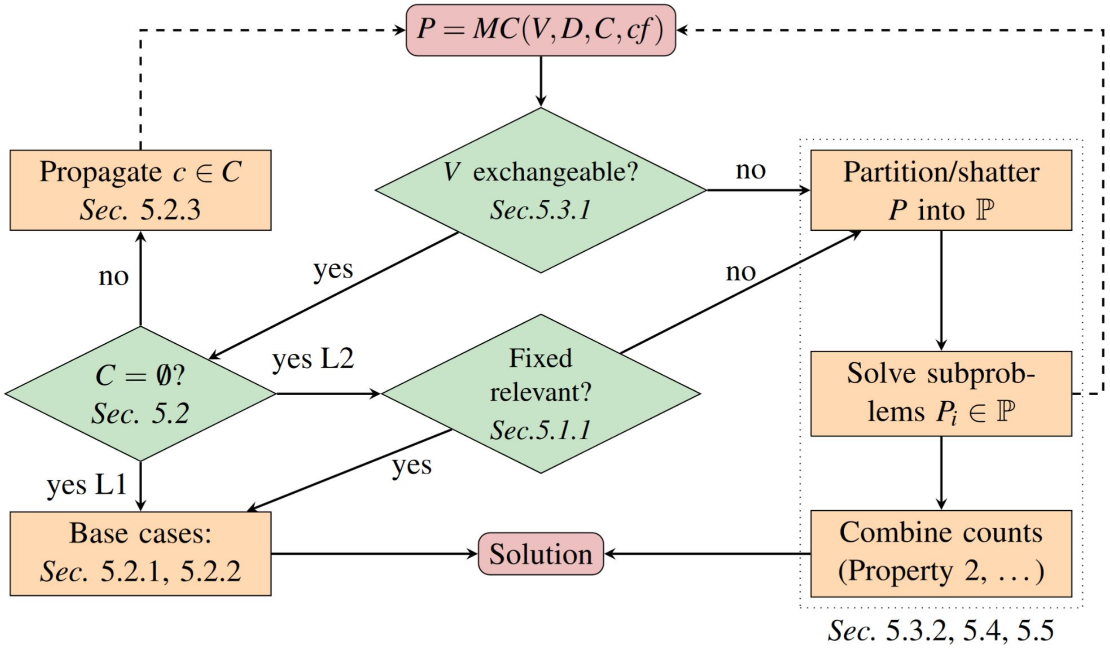
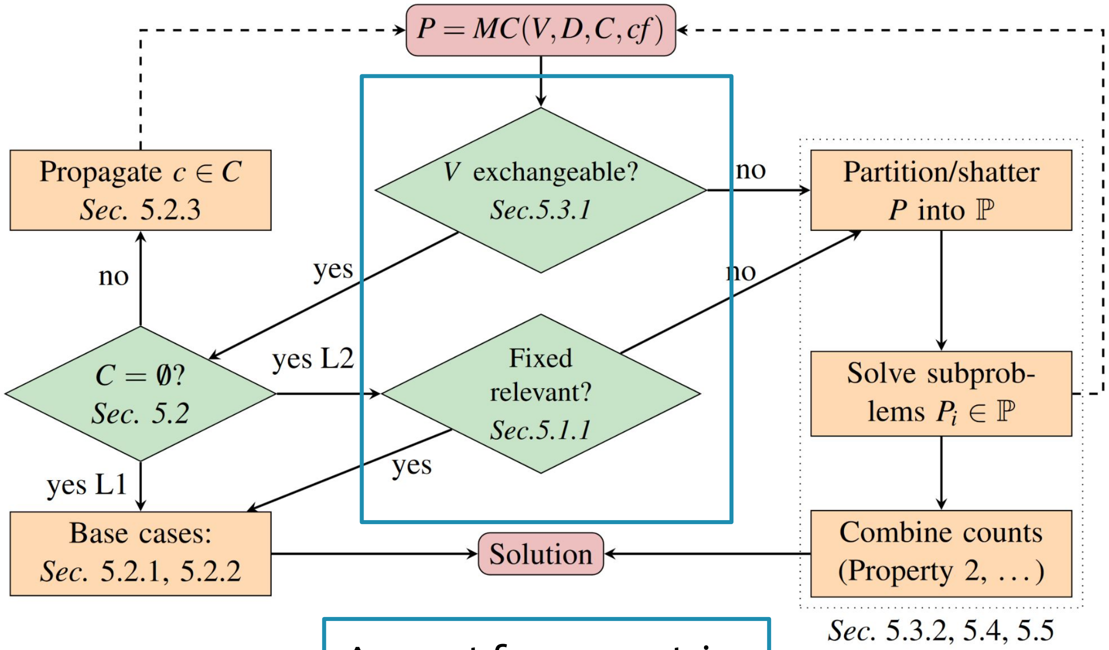
CoSo: a lifted Combinatorics problems Solver



Example 2

"A shipment of 12 different TVs contains 3 defective ones. In how many ways can a hotel purchase 5 of these TVs and receive at least 2 of the defective ones?"

CoSo: a lifted Combinatorics problems Solver



Account for symmetries

Example 2

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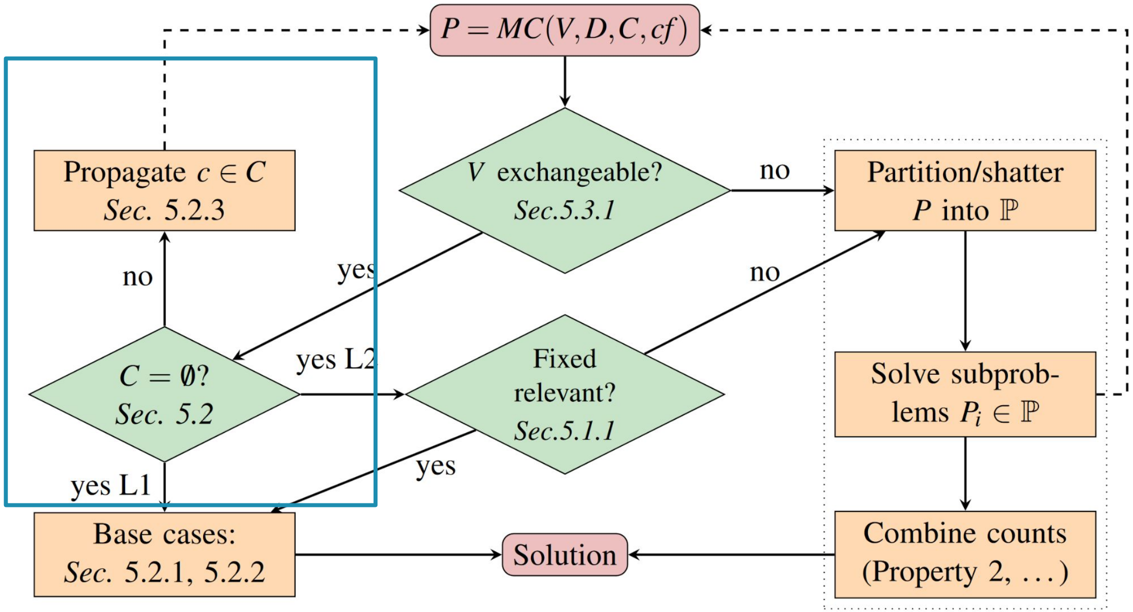

Propagate constraints

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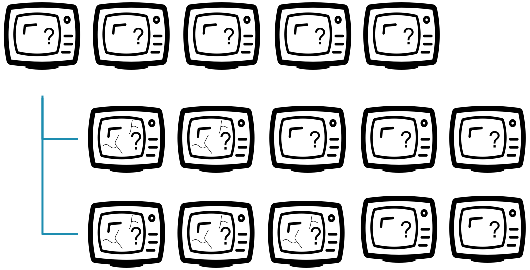

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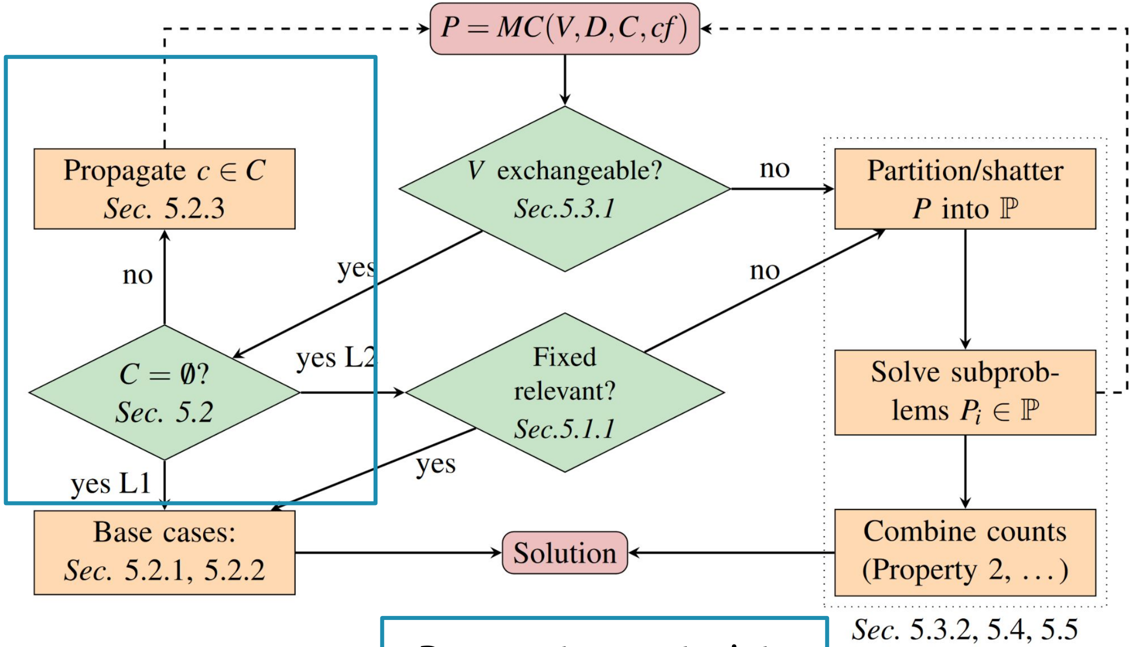

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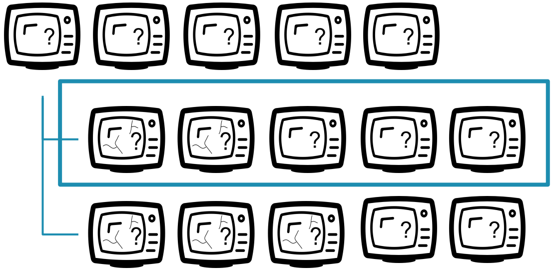

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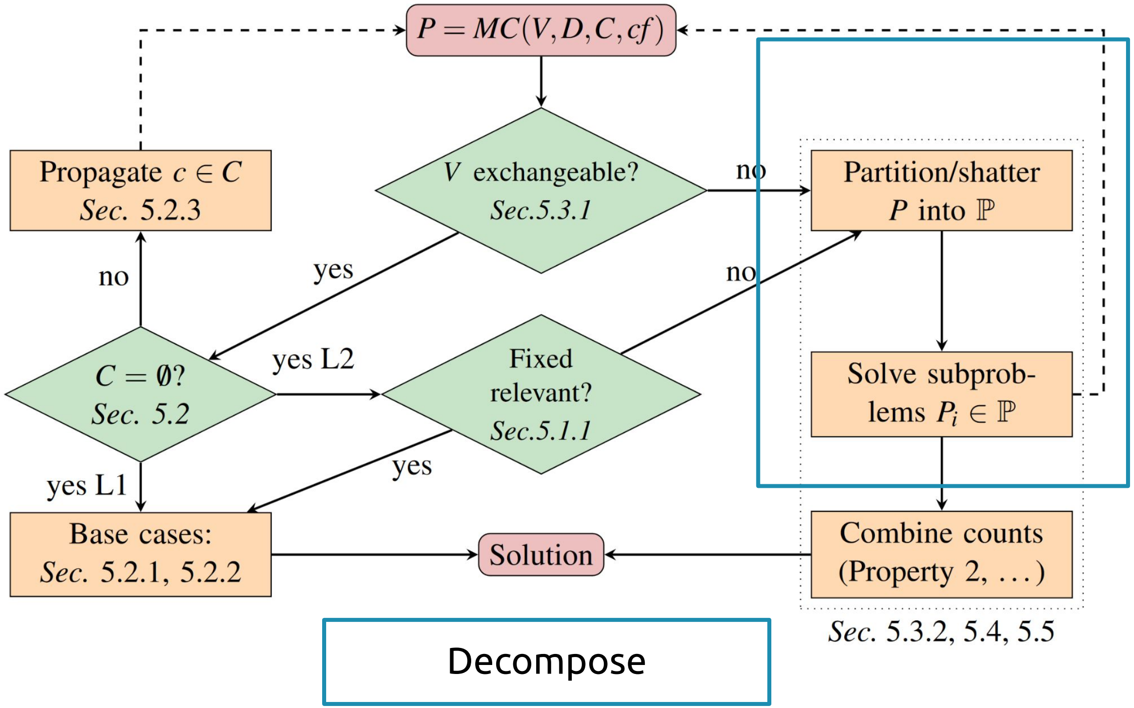

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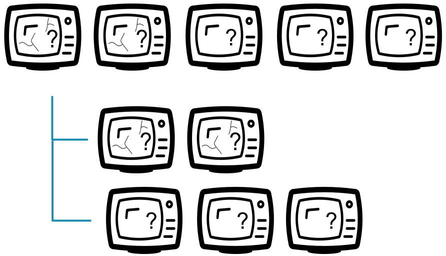

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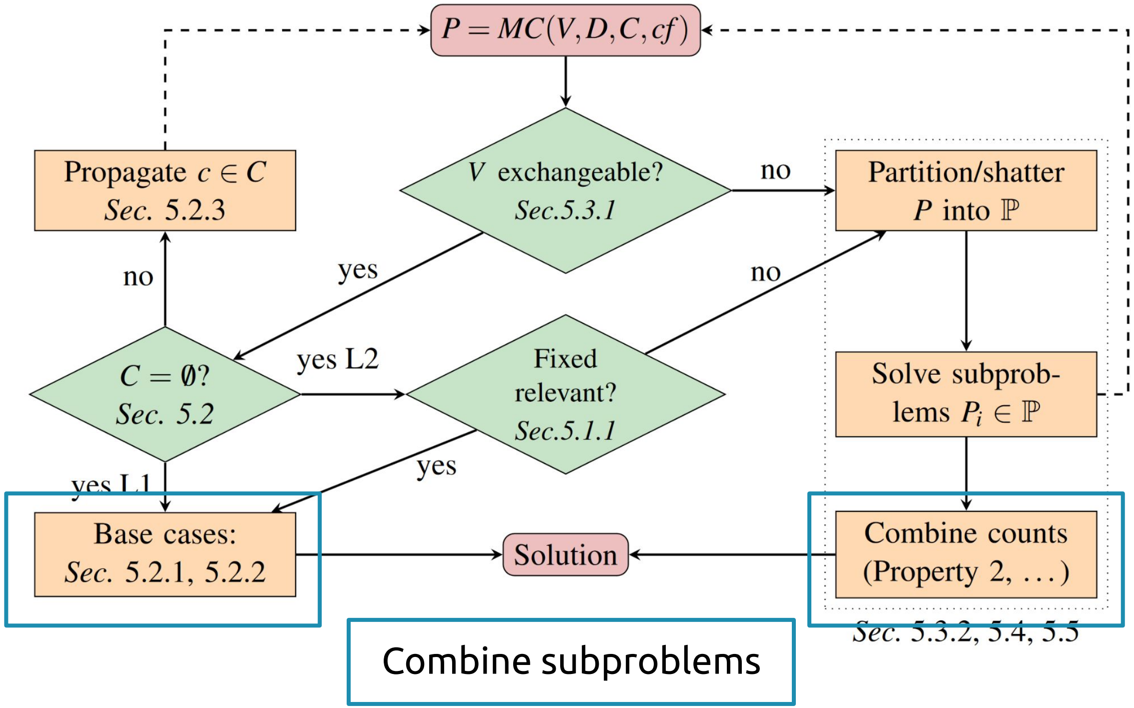

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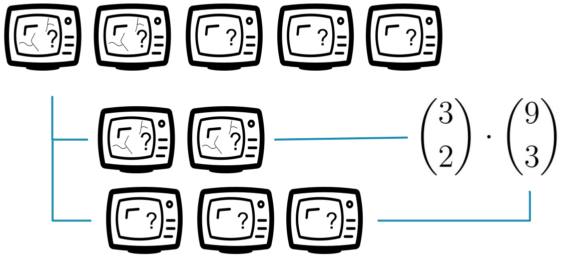

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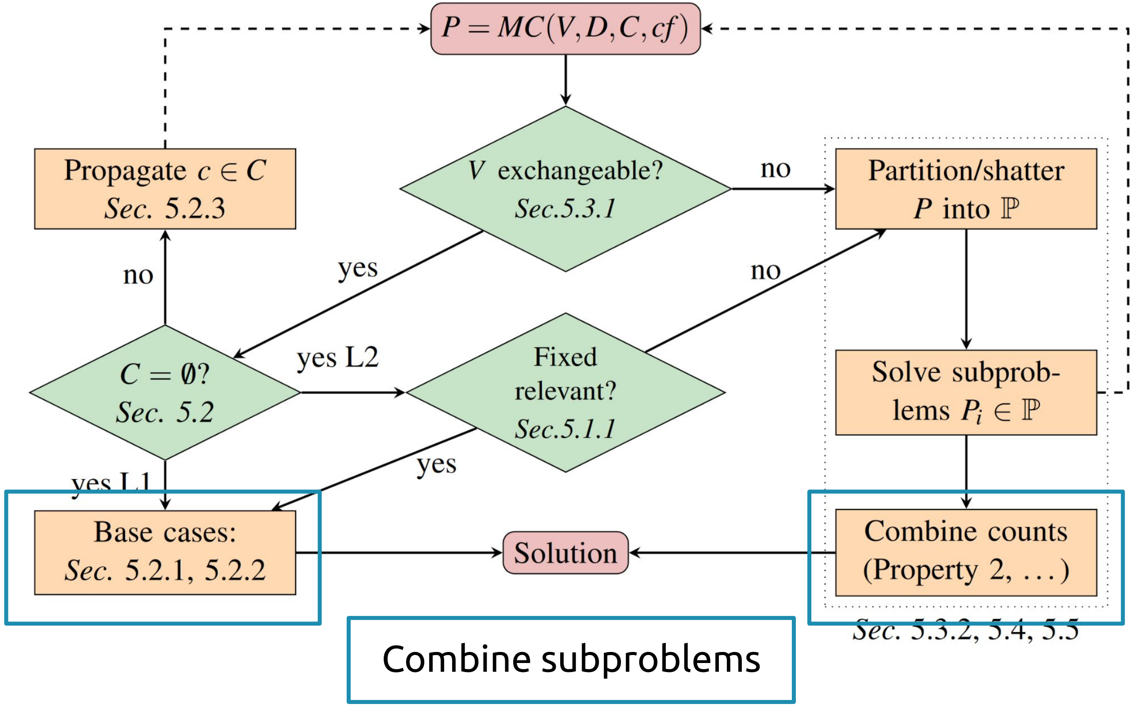

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$$\binom{3}{2}\cdot\binom{9}{3}+\binom{3}{3}\cdot\binom{9}{2}=288$$

Contribution 2: CoSo solver

Example 1	Contribution 1	Contribution 2	Result
"In how many different ways can the letters in B A N A N A be written?"	`universe letters = {a,a,a,n,n,b}; word in [letters]; #word = 6;`	$\frac{6!}{2! \cdot 3!} =$	60

Example 2	Contribution 1	Contribution 2	Result
"A shipment of 12 different TVs contains 3 defective ones. In how many ways can a hotel purchase 5 of these TVs and receive at least 2 of the defective ones?"	`labelled property tvs; property defective; #tvs = 12; #( tvs & defective ) = 3; purchase in { tvs }; #purchase = 5; #( purchase & defective )>=2;`	$\binom{3}{2} \cdot \binom{9}{3} + \binom{3}{3} \cdot \binom{9}{2} =$	288

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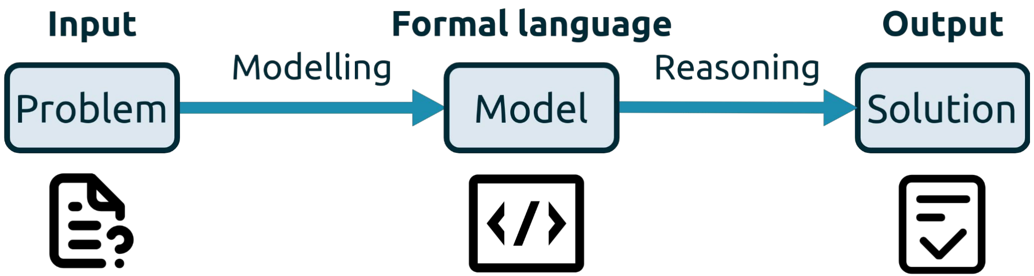
Combinatorial Problem Solving



Problem

Compute the solution of a combinatorics math problem: given a set of objects find the number of valid configurations defined.

Components

Key points to define:

1. Formal language (CoLa)
2. Reasoning (CoSo)

Output

The count of valid configurations (*model counting*).

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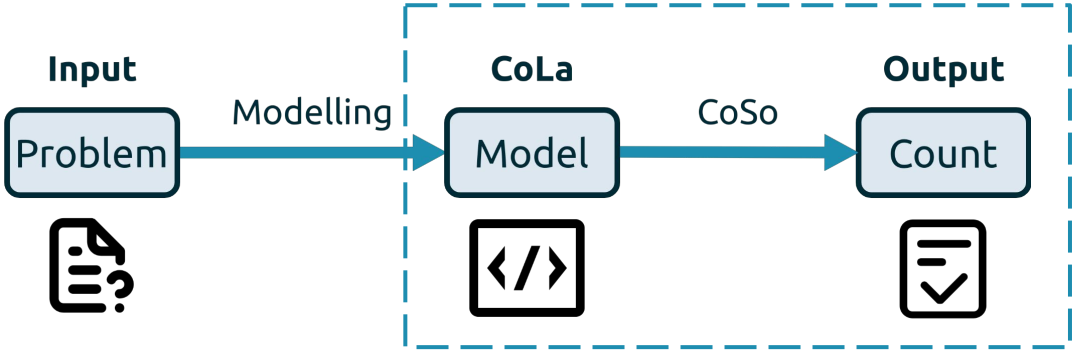

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Combinatorial Problem Solving

	Sets/multisets	Configurations	Constraint arity	Modelling	Lifted counting
Forclift	x	x	=2	x	✓
WFOMC FO^2	x	x	≥2	x	✓
GC-FOVE	x	x	≥2	x	✓
CSPs	sets	✓	≥2	✓	x
CoLa+CoSo	✓	✓	≥2	✓	✓

- Probabilistic frameworks implement lifted counting algorithms, but are based on first-order logic, with limited constraint support and limited input language

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GC-FOVE	x	x	≥2	x	✓
CSPs	sets	✓	≥2	✓	x
CoLa+CoSo	✓	✓	≥2	✓	✓

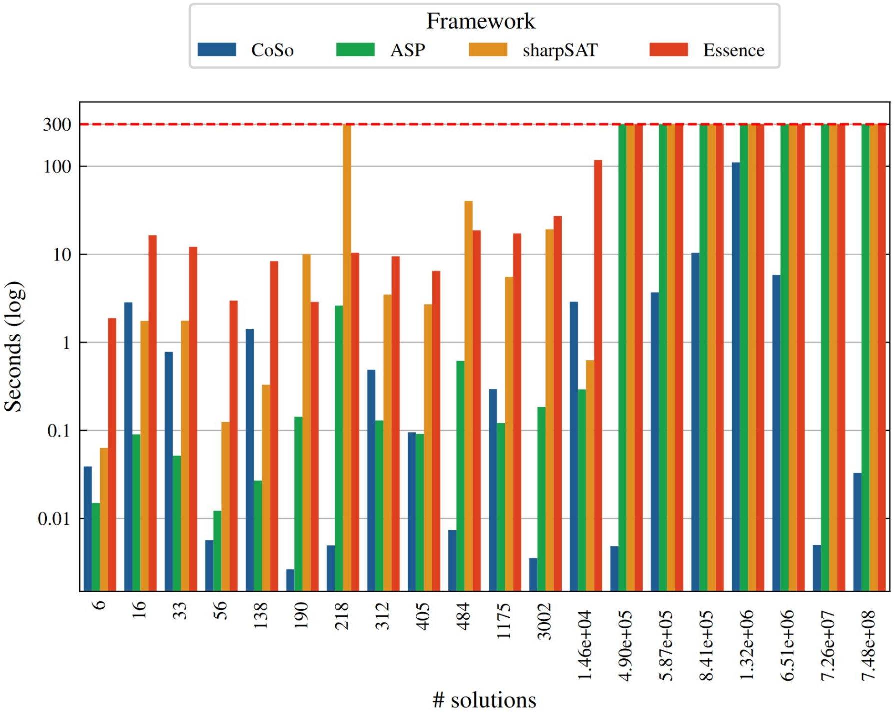
- CSP frameworks offer primitives for sets and constraints, but not for multisets, and counting is typically done via (propositional) enumeration of the solutions

Experiments

- CoSo outperforms propositional frameworks with the increase of the number of solutions

Real-world dataset	# unsolved /185 problems	avg. time (solved)
CoLa-CoSo	0	0.18s
ASP-Clingo	52	5.70s
CNF-sharpSAT	75	7.44s
ESSENCE-Conjure (CSPs)	32	34.99s

Synthetic benchmarks



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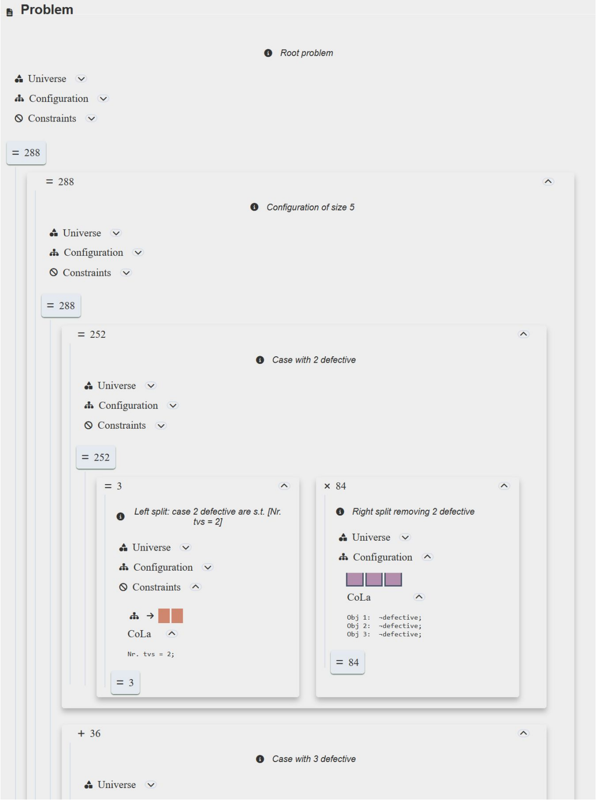
Resources

JAIR paper

https://github.com/PietroTotis/CoSo



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